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Comparison of the MacCormack's and the Kurganov–Tadmor's Schemes for Sod's Problem

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Abstract—The Sod's problem on the occurrence of a shock wave, a rarefaction wave, and a contact discontinuity in a pipe with different parameters on the left and right sides is solved numerically by two schemes: the MacCormack's method and the Kurganov–Tadmor's method. Comparison of graphs of gas density, velocity and pressure showed good agreement between solutions obtained by MacCormack's method and by Kurganov–Tadmor's scheme.

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1. SOD'S PROBLEM

In science and technology, resonators are widely used, which are pipes with a vibrating piston at one end. The study of the processes occurring in the resonator is complicated by the nonlinearity of these processes, so they are studied numerically. A number of works [1–4] are devoted to the numerical study of gas dynamics in resonators, in which various methods and approaches are used.

One of the benchmark problem in hydrodynamic, which solution validates various methods, is the Sod's problem [5]. The space $0 \le x \le L$ is filled with air at rest and is partitioned off by a membrane at $x = 0.5L$, on both sides of which pressure and density are set arbitrarily. At initial moment the membrane is removed, as a result of which the following instantly arise: a shock wave traveling to the right, a rarefaction wave traveling to the left, and a contact gap.

For Sod's problem the parameters in the left domain ($0 \le x < 0.5L$) were set equal to $\rho_1 = 1.29$ kg/m^3 (density), $u_1 = 0$ m/s (velocity), $p_1 = 109511.71$ Pa (pressure); in the right domain ($0.5L < x \le L$) were set equal to $\rho_2 = 0.125\rho_1$, $u_2 = 0$ m/s, $p_2 = 0.1p_1$.

2. SOLUTION OF THE PROBLEM BY THE MACCORMACK'S METHOD

Gas dynamics is described by hyperbolic equations

$$\frac{\partial \mathbf{w}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{w})}{\partial x} = 0,$$

where $\mathbf{w} = \{\rho, \rho u, e\}^T$ is a vector of conservative variables and $\mathbf{F}(\mathbf{w}) = \{\rho u, \rho u^2 + p, u(e+p)\}^T$ is flux vector. To close the system, we write the equation for pressure for perfect gas

$$p = (\gamma - 1)(e - 0.5\rho u^2).$$

After the introduction of uniform mesh $x_i = i\Delta x$ ($i = 0, ..., N$), $t^n = n\Delta t$ ($n = 0, 1, 2, ...$) with grid step Δx and time step Δt, one gets vector of state $\mathbf{w}_i^n$ at node i at moment t^n.

The MacCormack's scheme [5] consists of two steps: predictor: $\mathbf{w}_i^* = \mathbf{w}_i^n - (\Delta t/\Delta x)(\mathbf{F}_{i+1}^n - \mathbf{F}_i^n)$; corrector: $\mathbf{w}_i^{n+1} = 0.5(\mathbf{w}_i^n + \mathbf{w}_i^*) - 0.5(\Delta t/\Delta x)(\mathbf{F}_i^* - \mathbf{F}_{i-1}^*)$, where $\mathbf{F}_i^n = \mathbf{F}(\mathbf{w}_i^n)$, $\mathbf{F}_i^* = \mathbf{F}(\mathbf{w}_i^*)$.

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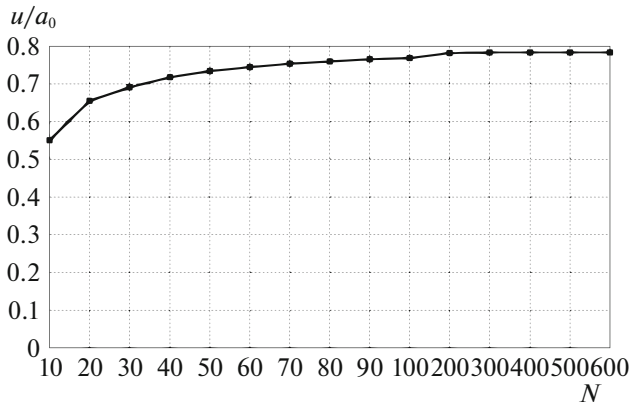

Fig. 1. Grid convergence of rhoCentralFoam solver.

3. SOLVING THE PROBLEM WITH OPENFOAM

OpenFOAM always operates in a 3 dimensional Cartesian coordinate system and all geometries are generated in 3 dimensions. OpenFOAM solves the case in 3 dimensions by default but can be instructed to solve in 1 dimension by specifying a special `empty` boundary condition on boundaries parallel to the x-axis for which no solution is required. The computational domain is a rectangular parallelepiped of length $L = 1$ m.

To solve the Sod's problem using OpenFOAM, it is necessary to create the setFieldsDict file [6], which, after using the `setFields` utility, will set the initial distributions of pressure, velocity and temperature over the area. However, by formulation of the problem, the density distribution is given, not the temperature distribution. Therefore, it is necessary to calculate the temperature from the equation of state for an ideal gas. For the left domain, the equation of state will be written $p_1 = \rho_1 R T_1$, for the right domain: $p_2 = \rho_2 R T_2$. Since $R = c_p - c_v$, and $\gamma = c_p/c_v = 1.4$, then $R = c_p - c_p/\gamma = 286.8$ m/(s^2 K) and $T_1 = p_1/(\rho_1 R) = 296$ K, $T_2 = p_2/(\rho_2 R) = 236.8$ K.

Approximation schemes were chosen as follows: d/dt—Euler; grad, div—Gauss linear; Laplacian—Gauss linear corrected.

After preprocessing, which consists of creating a mesh with the `blockMesh` utility and setting the pressure, velocity, and temperature fields with the `setFields` utility, the calculation is made by the `rhoCentralFoam` solver. RhoCentralFoam is a density-based compressible flow solver based on central-upwind schemes of Kurganov and Tadmor [7].

4. RESULTS

Calculations of the gas parameters are carried out for various grid partitions. In Fig. 1 are depicted values of u/a_0 at $x = 0.5L$ for a number of grid nodes $N = 10$, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 300, 400, 500, 600. It can be seen that when choosing a grid $N \geq 200$, the velocity value practically does not change.

In Fig. 2 a graph of the time spent on the calculation using the OpenFOAM program is depicted. A linear dependence of the counting time on the number of grid nodes is noticeable.

The following parameters were set: the length of the computational domain $L = 1$ m, the number of nodes of the uniform grid is $N = 600$, control time $t = 519$ μ s. Figures 3–5 show plots of dimensionless density, velocity and pressure, where $\rho_0 = \rho_1 = 1.29$ kg/m^3, $a_0 = \sqrt{\gamma R T_1} = 344.75$ m/s, $p_0 = p_1 = 109511.71$ Pa. The graphs calculated by the MacCormack's method and using the OpenFOAM program are in good agreement and completely matches in undisturbed areas, shock wave and rarefaction wave. Graphs calculated using OpenFOAM have a steeper head of rarefaction wave (Figs. 3–5) and a steeper contact discontinuity (Fig. 3), but weak oscillations are visible between the contact discontinuity and the shock wave. The graphs calculated by the MacCormack's method do not have oscillations, however, they are more smeared near the head of the rarefaction wave and near the contact discontinuity.

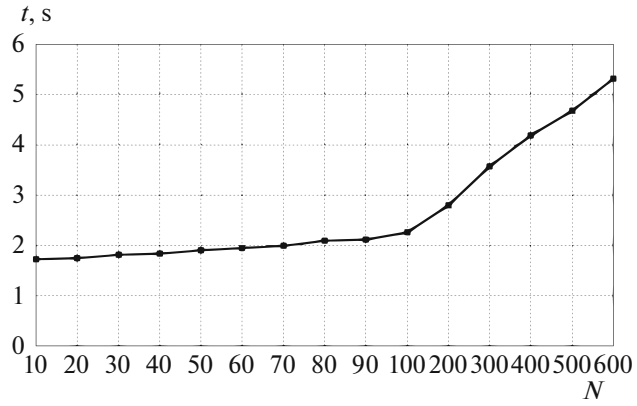

Fig. 2. Calculation time using OpenFOAM.

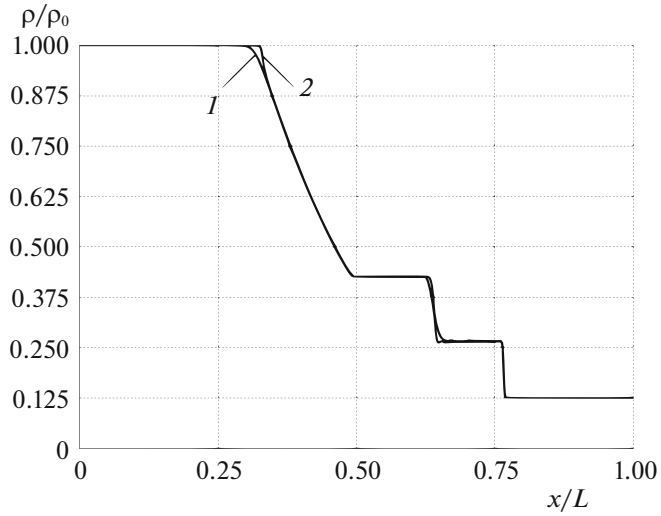

Fig. 3. Gas density. *1*—MacCormack's method, *2*—Kurganov and Tadmor's method.

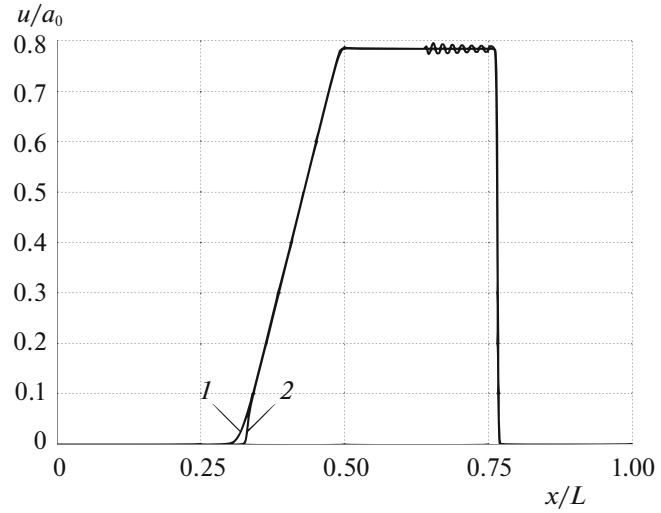

Fig. 4. Velocity. *1*—MacCormack's method, *2*—Kurganov and Tadmor's method.

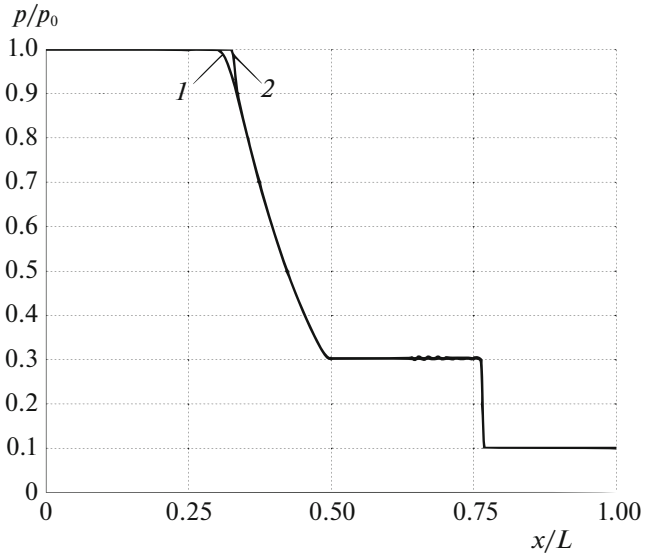

Fig. 5. Pressure. *1*—MacCormack's method, *2*—Kurganov and Tadmor's method.

5. CONCLUSIONS

The Sod's problem was solved numerically by the MacCormack's method and the finite volume method (OpenFOAM program, rhoCentralFoam solver for calculating compressible flows). Graphs of the gas density, velocity and pressure are compared; a good agreement between results was observed. OpenFOAM calculation showed small oscillations between the shock wave and the contact discontinuity. In graphs, calculated by the MacCormack's method, the head of the rarefaction wave and the contact discontinuity are smeared more than that in graphs, calculated by OpenFOAM.

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